MATHEMATICS D

Paper 4024/11
Paper 1

Key messages

In order to do well in this paper, candidates need to have covered the entire syllabus and have accurately learned the necessary formulae. They should be able to deal competently with basic arithmetic and produce accurate clear diagrams. They need to be able to select a suitable strategy for solving the more complex mathematical problems.

It is important to read the questions on the paper carefully, to highlight the key points and the form in which the answer is required, and to give the result to a suitable degree of accuracy.

General comments

This is a non-calculator paper and requires accuracy in basic number work. Some candidates need to improve on their computational skills in order to gain more marks.

In general, candidates performed well on number and standard algebra questions. Areas of weakness include relative frequency, histograms, bearings and vectors. Candidates should be able to recognise angles identified by three letters, such as angle *ABC*, as well as those identified by a single letter, such as *x*.

Good arithmetic skills were evident in many cases. Some candidates would benefit from checking their answers, in part to ensure sensible answers. It was common to see an incorrect answer resulting from a correct method involving arithmetic slips, particularly where negative numbers were involved.

When a question asks for an answer in its simplest form, candidates should be aware that an unsimplified answer will not gain full credit.

Presentation of the work was usually good with most candidates showing clear and sufficient working. This should be shown in the answer space next to the question. Candidates should be reminded that, when they replace work, they should cross it out clearly. They should not overwrite their answers, when they have made an error or if they have worked in pencil, as this can lead to illegible answers.

Candidates are advised to read the question very carefully, to ensure that they are answering the question being asked of them. This was particularly crucial in the question on direct proportion, where some candidates did not answer the question asked of them.

It is important that candidates note the instruction on the cover page that ‘the omission of essential working will result in loss of marks’ as opportunities to score some marks for steps within the working will be lost.

It is important that candidates write all numbers clearly – too often numbers are written hurriedly and are not formed correctly - in some cases it was difficult to distinguish between the digits 1, 2 and 7 and also 4 and 9.

Comments on specific questions

Question 1

(a) Most candidates answered this part correctly, with any errors arising from arithmetic slips.

(b) Most candidates obtained the correct answer, with any errors generally coming from either incorrectly converting a mixed number to an improper fraction or from calculating $\frac{2}{5}$ x $\frac{6}{5}$ as $\frac{12}{5}$.

Answers: **(a)** $\frac{17}{35}$ **(b)** $\frac{12}{25}$

Question 2

(a) Candidates who converted $17\frac{1}{2}$ per cent to $\frac{35}{200}$ and then simplified the fraction were generally successful. Common errors included answers left as $\frac{35}{2}$ (per cent) or $\frac{17\frac{1}{2}}{100}$. Some answers were cancelled down too far, for example to $\frac{3.5}{20}$ or $\frac{0.7}{4}$. A fractional answer in its simplest form must be $\frac{(\text{integer})}{(\text{integer})}$.

(b) Most candidates answered this correctly. There were a number of basic arithmetic errors and the common wrong answer of 6, from $6 + 4(0.6) = 10 \times 0.6$, was due to carrying out the operations in the wrong order.

Answers: **(a)** $\frac{7}{40}$ **(b)** 8.4

Question 3

Most candidates used the correct relationship between the variables and found the correct constant of proportionality, although some incorrectly found $\frac{16}{8}$ rather than $\frac{8}{16}$. A common incorrect answer was $\frac{3}{2}$ from using x, rather than x^2, in finding y when $x = 3$. Some candidates misread the question and used the relationship $y = kx$ or an inverse proportion such as $y = \frac{k}{x^2}$ or $\frac{k}{\sqrt{x}}$.

Answer: 4.5

Question 4

Although there were many correct answers, a significant number of candidates found $\frac{5}{30}(\times 100)$ rather than $\frac{5}{25}(\times 100)$, not realising that percentage increase is based on the original value rather than the new value. There were a few answers giving the increase of 5 rather than the percentage increase.

Answer: 20

Question 5

(a) Most candidates answered this part correctly, with any errors mainly coming from only partially factorising the expression.

(b) Most candidates answered this part correctly. Common errors were due to only partially factorising the expression or having one bracket as $(x-3)$ or $(3-x)$.

Answers: **(a)** $3a(5+b)$ **(b)** $(x+3)(2k-y)$

Question 6

(a) Most candidates obtained the correct answer, with the common wrong answer being $\frac{3}{2}$. Negative answers are best written with the negative sign in front of the number so –1.5 or $-\left(\frac{3}{2}\right)$ are preferable to $\frac{3}{(-2)}$.

(b) Many candidates were able to complete the first step of reaching $y(x+4)=3$ but were then unaware that the next steps were to multiply out the bracket and isolate the x term. Some candidates simply gave a numerical answer.

Answers: **(a)** –1.5 **(b)** $\frac{3-4x}{x}$

Question 7

(a) Most candidates answered this part correctly, with the common error being to find a term other than the first.

(b) Candidates found this part more challenging. A common incorrect response was $n+4$ as was $4n+k$, where k was not equal to 5. Candidates who used a formula to reach the result sometimes did not simplify their expression.

Answers: **(a)** 9 **(b)** $4n+5$

Question 8

(a) Generally candidates did not know the meaning of the word 'irrational' and often gave decimal or fractional answers. Those having some understanding of irrational numbers sometimes gave answers such as $\sqrt{(4.5)}$. It was rare to see answers involving π.

(b) There was often very little sign of any calculation of a division by 8 being carried out in order to find the answer. Common wrong answers were 5, 6, 8 and also 16.

Answers: **(b)** 4

Question 9

Many candidates had difficulty understanding the angle notation (with three letters) used in this question.

(a) Candidates who recognised that the angles at the point G totalled 360° mostly carried out an accurate calculation to reach the correct answer.

(b) Candidates recognising that the interior angles between the parallel lines AB and FG totalled 180° were usually successful.

(c) Candidates recognising that the alternate angles between the parallel lines HFE and GD were equal were usually successful.

(d) Candidates recognising that the interior angles between the parallel lines *HB* and *FG* totalled 180° were usually successful.

Answers: **(a)** 110° **(b)** 50° **(c)** 120° **(d)** 60°

Question 10

Most candidates understood the concept of a net and were able to correctly draw at least one of three required rectangles, usually drawing the two rectangles at the top and bottom of the given diagram and not the one at the right hand side. Common errors were to draw two 2×2 rectangles at the top and bottom or to draw a 3D diagram.

Answer: Correct net

Question 11

(a) Most candidates reached 16×10^{10} with many completing their answer to give a number in standard form. When errors occurred they were usually the result of incorrectly squaring 4 as 8 or giving an incorrect power of 10 by adding 5 and 2 instead of multiplying. Occasionally 16×10^{10} was rewritten as 1.6×10^{9}.

(b) Candidates who recognised that $\dfrac{1}{\left(4 \times 10^5\right)}$ needed to be split into $\dfrac{1}{4}\ \dfrac{1}{10^5}$ were usually able to reach $0.25 \times$ a power of 10. Some were then able to correctly convert this to standard form. Many answers were either given incorrectly as ($4 \times$ a power of 10) or left as a fraction.

Answers: **(a)** 1.6×10^{11} **(b)** 2.5×10^{-6}

Question 12

Many candidates were able to correctly write the given values to 1 significant figure and carry out the calculation. The common error was then the placement of the decimal point in the answer.

Some had difficulty in writing the numbers to 1sf with 614.2 being given as 614 or 60, and 0.0304 given as 0.0. Candidates should be aware that 600.0, 0.0300 and 20.00 are not numbers to 1 significant figure. Some attempts at long multiplication or long division were seen; this is never required in this type of question.

Answer: 600, 0.03 and 20 seen with final answer of 0.9 or $\dfrac{9}{10}$

Question 13

Candidates found this question demanding with a minority dealing correctly with both the bar widths and the heights. Some realised that the widths of the bars were not all the same and were given credit for this. There was a general lack of understanding that the heights of the bars should reflect the frequency densities and not the frequencies. Some answers were given incorrectly as a frequency polygon (but with correct heights). A histogram does not have vertical gaps between the bars.

Answer: Correct histogram with frequency densities 1, 1.5, 2, 2.4, 0.8

Question 14

(a) Many candidates were able to deal with both powers and reach the correct answer. Common errors included $\left(\dfrac{1}{3}\right)^0 = 0$ and $\left(\dfrac{1}{3}\right)^2 = \dfrac{1}{6}$. A few candidates worked out their answer as a decimal which, in this case, is not an exact value.

(b) This type of answer requires a logical approach with clear steps of working rather than random steps with no obvious strategy. Those who inverted the fraction to give $\left(\frac{x^6}{27}\right)^{\frac{1}{3}}$ and then found the cube root were usually correct. Common errors included giving the cube root of 27 as 9 and the cube root of x^6 as x^3. Some unsimplified answers were $\frac{1}{3x^{-2}}$ and $\frac{3^{-1}}{x^{-2}}$.

Answers: **(a)** $\frac{8}{9}$ **(b)** $\frac{x^2}{3}$

Question 15

(a) A minority of candidates gave the correct answer as there was a lack of understanding of the meaning of relative frequency with common wrong answers of 50 or 105 (from 30 + 25 + 50).

(b) Many candidates obtained the correct answer, usually by realising that 20 was $\frac{1}{10}$ of 200 so their answer would be a $\frac{1}{10}$ of 50. Although the spinner was spun only 20 times, there were a number of answers greater than 20.

(c) This question was not answered well by candidates, with many varied responses. Candidates did not understand the concept of **fair** in mathematical terms leading to incorrect responses about how the experiment was carried out. Candidates also thought that if the experiment was **fair** then all the frequencies should be equal rather than that they should be quite close to each other, (which they were not in this case). Some thought that as Ashraf carried out the experiment more often, that would make it fair. This only makes the result more accurate, not fairer.

Answers: **(a)** $\frac{50}{200}$ **(b)** 5 **(c)** No, with a supporting reason

Question 16

(a) There were a number of correct answers but many candidates did not recognise that the total distance travelled on the journey was from *P* to *Q* to *R* and then back again. As a result many answers were given as $\frac{15}{5}$. Others found the average speed for each part and then either added them together or found their mean value. A few candidates tried to include the angles shown on the diagram.

(b) (i) Candidates found part **(b)** demanding. Drawing in a North line at *Q*, and recognising that the bearing from this North line to the dotted line at *Q* is 040°, leads to 40° + 90° being the bearing of *R* from *Q*. Candidates could improve on this topic by remembering that all bearings are measured from a North line clockwise.

(ii) The bearing of *P* from *Q* can be found by starting at the North line at *Q* and finding 40° + 90° + 90° (angle *PQR*) = 220°.

Answers: **(a)** 6 **(b)(i)** 130° **(ii)** 220°

Question 17

(a) Most candidates answered this part correctly. Errors were generally arithmetic.

(b) Candidates found this part demanding and correct answers often came from trial and improvement. To reach a square number from $2^3 \times 3 \times 5$, the powers need to be even numbers, which gives the smallest value of n as $2 \times 3 \times 5$.

Answers: **(a)** $2^4 \times 3 \times 5^2$ **(b)** 30

Question 18

There were many correct answers from using the total interior angle sum of the hexagon as $(6 - 2) \times 180$, subtracting the given four angles and dividing the result by 2. Errors included arithmetic errors, not knowing $(n - 2) \times 180^\circ$ as the sum of the interior angles of a polygon and not realising that the total sum of all the interior angles had to be greater than 470°. A few candidates took the alternative approach by finding the four exterior angle from the given angles first. These candidates often forgot the final step of taking the exterior angle from 180°, to give the interior angle required, thus giving their answer as 55°, the exterior angle to 125°.

Answer: 125°

Question 19

(a) Many candidates answered this part correctly. Incorrect answers were usually due to inaccurate measuring or measuring the wrong angle.

(b) (i) Many candidates constructed the bisector correctly. Few tried to draw the angle bisector without using a pair of compasses or at the wrong vertex.

(ii) Many candidates used the correct method and obtained the correct line. A few drew a line which was too short, did not cross *AB* or used only one pair of intersecting arcs. Occasionally the candidate became confused over which points of intersection of pairs of arcs belonged to which part of the question and joined incorrect arcs.

Answers: **(a)** 96° to 98°

Question 20

(a) Most candidates obtained the correct answer with a few making arithmetic errors, often in finding $2 - (-2)$, or miscopying their answer from the working space to the answer line. A small number tried to carry out a multiplication of matrices.

(b) Many correct answers were seen. It is acceptable to leave the $\frac{1}{2}$ outside the matrix – errors were sometimes made when attempting to simplify their answer. On occasion, the value of $2 \times 1 - (-1) \times 0$ was given as 3. A few candidates showed a lack of understanding of the meaning of $\mathbf{A}^{-1}$ and found the reciprocal of all four numbers or wrote $\frac{1}{\mathbf{A}}$ as their answer.

Answers: **(a)** $\begin{pmatrix} 0 & -5 \\ -6 & 4 \end{pmatrix}$ **(b)** $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$

Question 21

(a) The most common correct inequality was $x > 2$. A number of candidates correctly rearranged $6x + 7y < 42$ to make y the subject of the inequality, although this rearrangement was not required. Some candidates knew what was required but used an incorrect inequality sign. There were a few occasions when $y = \frac{x}{5}$ was used.

(b) This was a challenging question for most candidates and was rarely fully correct. Few recognised that finding either the coordinates of C or the gradient of OC would give them the values between which k would lie. Very few candidates showed any working and a number of candidates did not attempt the question.

Answers: **(a)** $x > 2$ and $6x + 7y < 42$ **(b)** 1 and 2

Question 22

(a) Some candidates obtained the correct answer with others identifying the reflection but not the line of reflection. The common error was to give the answer as a rotation. Within the description, there should be no mention of centres, anticlockwise or similar statements. A few candidates failed to read the instruction to give a single transformation.

(b) Candidates generally drew either the correct rotation or the rotation about a different centre. A few rotated shape A anticlockwise in error.

(c) Although many candidates knew that the answer was a 2×2 matrix containing 1 or –1 with 0's, the correct matrix was rarely seen. Finding the new position of the unit vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ can identify this matrix. A number of candidates gave a 2×2 or a 2×3 matrix related to the coordinates of either the original or new shape, or did not attempt this part.

Answers: **(a)** Reflection in $y = -x$ **(b)** Triangle with vertices (1, 0), (3, 0) and (3, 1) **(c)** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Question 23

Candidates find vectors a demanding topic. They could improve their skills by reading the question carefully as it will indicate in which form an answer is required. For this question, all answers were to be in terms of **p** and/or **q**.

There were a number of candidates who did not attempt the whole question.

(a) Candidates needed to use the ratio given at the top of the question to realise that BX was $\frac{3}{4}$ of BC leading to $\frac{3}{4}$ of 8**q**.

(b) The vector required can be found by following the route from A to B to X. A number of candidates were able to add 6**p** to their answer in **(a)**.

(c) (i) The required vector can be found by following the route A to D to C to Y.

(ii) Candidates who successfully answered the earlier parts to the question were then able to reach the ratio $6(\mathbf{p} + \mathbf{q}) : 9(\mathbf{p} + \mathbf{q})$ leading to the answer. The answer is numerical and not in terms of vectors.

Answers: **(a)** 6**q** **(b)** 6**p** + 6**q** **(c)(i)** 9**p** + 9**q** **(ii)** 2:3

Question 24

(a) Many candidates correctly found $\frac{(44-20)}{10} = 2.4$ but some then went on to halve this answer, not realising that the deceleration does not change during that 10-second time interval, even though $t = 5$ is halfway between $t = 0$ and $t = 10$.

(b) Candidates who realised that, as 5 was halfway between 0 and 10, the speed was halfway between 20 and 44, were able to correctly calculate the required speed.

(c) The area of the trapezium on the left hand side of the diagram was often correctly calculated, although some answers did not include the rectangle at the base. Candidates found it more difficult to find the area on the right – a rectangle with sides 20 and k giving the area as $20k$. Common errors for this area were to lose the brackets, to write $10 + k - 10 \times 20$, or to use $(10 + k) \times 20$.

Answers: **(a)** 2.4 **(b)** 32 **(c)** 16

Question 25

(a) Candidates who recognised that, in similar triangles, it is the sides of the triangles that are compared, were able to write down $\frac{AP}{AB} = \frac{PQ}{BC}$. They generally then reached $PQ = \frac{36}{8}$ and many were then able to cancel this correctly to the correct answer of 4.5. The common error was to have $\frac{3}{5}$ rather than $\frac{3}{8}$ in the equation to find PQ, thus leading to the incorrect answer of 7.2.

(b) Candidates found this part very demanding. There were few correct answers and many omitted this part. Candidates can improve on this topic by remembering that the linear scale factor has to be squared when finding the area scale factor. Few candidates reached $x - \left(\frac{3}{8}\right)^2 x$ or even gave an answer totally in terms of x.

Answers: **(a)** 4.5 **(b)** $\frac{55}{64}x$

MATHEMATICS D

Paper 4024/12
Paper 1

Key messages

In order to do well in this paper, candidates need to

- have covered the whole syllabus
- remember necessary formulae and facts
- recognise, and carry out correctly, the appropriate mathematical procedures for a given situation
- perform calculations accurately
- show clearly all necessary working in the appropriate place.

General comments

Presentation of work was usually good. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. Working makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results.

There were some scripts where candidates had not attempted many questions and had made poor attempts at the others. Some candidates did not appear to bring geometrical instruments into the examination.

Some candidates were not prepared for questions on the new syllabus topics of correlation and nets.

At times, candidates appeared not to read the questions carefully. They should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer.

Questions that proved to be particularly difficult were **11(c)**, **13(a)**, **17(b)**, **19(a)**, **20(c)** and **22(c)**.

It was noticeable that a significant number of candidates need to improve their ability to approximate and to use appropriate degrees of accuracy; and also to understand the integer class of number. Some candidates were very competent at performing standard techniques, and yet seemed unable to recognise the appropriate mathematical procedure required for a given situation.

Candidates need to improve their skills in manipulating fractions and in basic arithmetic, particularly when negative numbers are involved. Others need to improve their knowledge of the basic multiplication tables. Work that used correct methods was sometimes spoilt by wrong calculations.
It was also noticed that, quite frequently, a fraction such as $\frac{5}{10}$ was evaluated to 2.

A few candidates did not heed the instructions on the front page – to write in dark blue or black pen. Except for diagrams and graphs, candidates must not write in pencil. Nor should they overwrite pencil answers in ink, as this makes a double image which can be difficult to read. It is also an unnecessary waste of time. Some candidates wrote in pencil and even erased their workings, often producing rubber and paper debris that interfered with the clarity of other answers, particularly those with a decimal point and a negative sign.

Candidates should be made aware that only their final answer to each question should be written in the answer space. Alternative offerings and working should not be written there. When an answer is to be changed, it is far better to delete and replace the original one rather than attempt to write over it. Care must always be taken to ensure that answers obtained in the working are accurately transferred to the answer space.

Comments on specific questions

Question 1

(a) Most candidates added the fractions correctly, using 40 as the common denominator. Common errors were incorrect addition of the numerators or adding the numerators and denominators separately without the use of a common denominator. A few candidates made errors in unnecessarily converting a correct fraction to a decimal.

(b) Many candidates reached a correct answer in this part, with a few giving the answer as a fraction which was also acceptable. Some candidates multiplied the digits correctly but then made place value errors to reach answers such as 0.44, 0.044 or 44. A few candidates obtained the fraction $\frac{44}{10000}$ and went on to convert it incorrectly to a decimal. Occasionally the answer was given to 1 significant figure instead of the exact answer.

Answers: **(a)** $\frac{21}{40}$ **(b)** 0.0044

Question 2

(a) Attempts at this part were rather variable, and sometimes very lengthy. A short method was to realise that there are five portions of 200 grams in one kilogram, so the cost, in cents, is 5×85. Conversion to dollars was usually accomplished successfully. The common wrong answers were 425, 42.5 or 8.50

(b) This part was usually answered correctly. The common wrong answer was 12, from $\frac{60}{(2+3)}$.

Answers: **(a)** 4.25 **(b)** 40

Question 3

Candidates need to read this type of question very carefully, in order to be certain that they are answering the question being asked. Some candidates used *directly* proportional to *x*. Of those candidates who did answer the right question, there was a high percentage of fully correct answers. Others, having obtained $k = 90$, incorrectly, from the equation $30 = \frac{k}{\frac{1}{3}}$, usually went on to give $y = 18$.

A few candidates replaced $\frac{1}{3}$ by 0.3, or 0.33, and went on to obtain an answer which was not the exact correct answer.

Answer: 2

Question 4

Candidates attempted this question sensibly, usually by converting each fraction to a decimal, or to fractions with a common denominator, or to a percentage. Errors were often made in the conversion to decimals, particularly with the fraction $\frac{21}{25}$. Some candidates attempted to divide the numerator into the denominator.

A few thought that $\frac{4}{5}$ was the largest fraction in the list.

Answer: $\frac{4}{5}$ $\frac{41}{50}$ $\frac{21}{25}$ $\frac{17}{20}$ $\frac{9}{10}$

Question 5

(a) Many candidates answered this question correctly. Candidates who made errors did so by either failing to expand the brackets correctly, and getting $-2c - 15$, or by expanding $(4c - 3)(2c - 5)$. A few attempted to solve an equation.

(b) Most candidates answered this part correctly. If the final answer was not correct, a correct partial factorisation such as $2(4 - 5y)$ was usually seen. Those who tried to reorder the four terms frequently made a sign error.

Answers: **(a)** $15 - 2c$ **(b)** $(2 + 3x)(4 - 5y)$

Question 6

(a) This part was usually answered correctly, with $\frac{1}{6}$, or $\frac{-1}{-6}$, being the common wrong answers.

(b) This part of the question was found to be more challenging. Although most candidates were able to show the usual correct first steps by replacing $f(x)$ with y and obtaining $3xy = 2x - 5$, few could progress to $x(3y - 2) = 5$. It was common to see working such as $3x - 2x = -\frac{5}{y}$, and sign errors were sometimes seen in the rearrangement of the equation. A few answers were left in terms of y.

Answers: **(a)** $-\frac{1}{6}$ **(b)** $\frac{5}{3x-2}$

Question 7

Many candidates obtained the numbers 16 (from $22 - 6$) and 4 (from $10 - 6$) and used these to obtain the correct answer.

Many correct solutions were seen. These very often resulted from a correctly completed Venn diagram with 16, 6 and 4 in the correct subsets followed by a subtraction of 26 from 35.

A few candidates forgot the intersection and gave 15 as their response.

There were many answers of 3, these being the outcome of a variety of incorrect methods. The most common was the solution of the equation $22 - x + 6 + 10 - x + x = 35$ with '$22 - x$', '6', '$10 - x$' and 'x' shown on a Venn diagram.

Another quite popular approach emanated from entries of 22, 6 and 10 in the Venn diagram followed by $22 + 10 + 6 = 38$ and the subsequent subtraction $38 - 35 = 3$. Others who worked from the same Venn diagram but left out the intersection offered $35 - (22 + 10) = 3$.

Answer: 9

Question 8

(a) Most candidates realised that it was necessary to add three more flags and these were often correctly placed. However, a significant number drew the figure having rotational symmetry of order 2 and a small fraction opted for order 8.

Sometimes the drawn diagram had one, or two, lines of symmetry and no rotational symmetry.

(b) Many candidates answered this part correctly. Others can improve on this topic by working with lines of symmetry which are not horizontal or vertical.

Question 9

This question was generally answered well by those who know what is meant by standard form. Some candidates need to learn that standard form means $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer.

Difficulties were encountered in coping with the numerical calculation, the indices and the conversion to standard form.

(a) This part was well answered. The usual wrong answer was 16×10^5, which had not been converted to standard form as required, and 1.6×10^4.

(b) This part was quite well answered. The usual error was in the power of 10, with 5, –5 and 17 being the most common.

(c) There was much less success in this part. The most frequent wrong answers were 2×10^{-6} or 8×10^{-3} .

Answers: **(a)** 1.6×10^6 **(b)** 4×10^{-17} **(c)** 2×10^{-2}

Question 10

Candidates who began by rounding the given numbers correctly to 1200, 300 and 25 often reached the correct answer. Some had difficulty correcting to 2 significant figures, with 12 or 1212 sometimes used in place of 1200; and 30, 290 or 299 in place of 300. Occasionally $\frac{4}{5}$ became 1.25.

Answer: 0.8

Question 11

Candidates need to be aware that the formula $a + (n - 1)d$ for the nth term of a sequence only holds if the difference between *all* consecutive terms is the same value. This formula can be applied to the numerators and denominators separately but not to the fractions in the given sequence.

(a) This part was correctly answered by almost every candidate.

(b) Candidates who were guided by their answer to part **(a)**, and realised that the denominators are the multiples of 4, were able to deduce that the denominator of 1200 is the 300th term. Many tried to use $\frac{1199}{1200}$ in a variety of unsuccessful methods.

(c) A few candidates, having noticed that the denominators are multiples of 4 and each numerator is 1 less than its denominator, were able to write down the correct answer. Others separated the numerators and denominators and used the formula $a + (n - 1)d$ correctly. Many candidates tried, inappropriately, to use the formula $a + (n - 1)d$ with $a = \frac{3}{4}$ and $d = 1$.

Answer: **(a)** $\frac{23}{24}$ $\frac{27}{28}$ **(b)** 300 **(c)** $\frac{4n-1}{4n}$

Question 12

A few candidates used the exterior angles property and $\frac{360}{180-175}$ to obtain the correct answer directly. Occasionally, however, this expression was evaluated as 75.

Some knew the formula for the sum of the angles in a polygon and applied it correctly to reach the correct answer. Others used incorrect results such as $(n-2)\times 180 = 175$ or $n-2\times 180 = n-360$.

Answer: 72

Question 13

(a) Answers showed that many candidates need to improve their understanding of ratios of quantities given in different units and to be able to convert 1 kilometre to centimetres. Wrong answers such as $\frac{1}{2}$ km, 1:2, 1:500, or no attempt were very common.

(b) Answers showed that many candidates need to improve their understanding of bearings. The usual, wrong answer was 073°, although some added the 73 to, or subtracted 73 from, 90. A significant number believed that they needed to measure the length of *AB*.

(c) There were many good attempts with intersecting arcs of radii 5 cm and 6 cm. Occasionally the point of intersection was not identified as *C*, nor joined to *A* and/or *B*. Some produced the line from *B*, through the point of intersection of the correct arcs, until it met the North line from *A*, and labelled this point *C*.

Answers: **(a)** 50 000 **(b)** 286° to 289°

Question 14

Some candidates showed that they need to improve their knowledge of nets.

(a) Many candidates realised that one triangle and one rectangle were needed to complete the net. Whereas the size and position of the triangle were often correct, the size of the drawn rectangle was frequently 4 cm by 4 cm, or 4 cm by 3 cm. Errors demonstrated that candidates had not considered how the edges of the net would join when folded to make the prism.

(b) Attempts at this part were very varied. Many candidates seemed to have difficulty in visualising the five faces and did not use the net as a guide. Sometimes the individual areas were calculated correctly and then added incorrectly. Common errors were to omit the area of one of the rectangles; to calculate the area of a triangle as $\frac{1}{2}\times 5\times 3$ or $\frac{1}{2}\times 5\times 4$; to calculate the total length of all the edges or to calculate the volume of the prism.

Answers: **(b)** 60

Question 15

Many candidates showed that they need to improve their knowledge of correlation and lines of best fit.

(a) A few candidates omitted this part. Most plotted all the points accurately.

(b) This part was answered correctly only by a minority of candidates. Some of the incorrect answers were ‘linear’, ‘non-linear’, ‘variable’, ‘decreasing’, ‘fluctuating’, ‘inverse’ and ‘deceleration’.

(c) Many did not attempt to draw a line of best fit and only joined the plotted points, creating a zigzag line.

(d) Many candidates were able to correctly answer this part from their line.

Question 16

Candidates who started by writing the double inequality $15 < 2x - 3 < 22$ into the two separate inequalities $15 < 2x - 3$ and $2x - 3 < 22$ usually went on to get $x > 9$ and $x < 12.5$. However, many gave this as their final answer instead of giving the three integers between 9 and 12.5.

Candidates need to be confident of the meaning of the word 'integers'.

Common errors were to get $6 < x$ from $18 < 2x$; $x < 17.5$ from $2x < 25$ or $x < 9$ from $9 < x$.

Some candidates equated $2x - 3$ to each integer between 15 and 22 and solved these equations for x, though sometimes forgetting to reject the solutions which were not integers.

A sizeable minority were unable to deal with the initial double inequality and started from a variety of wrong inequalities such as $2x - 3 < 22 - 15$.

Answer: 10 and 11 and 12

Question 17

Some candidates showed that they need to improve their understanding of tree diagrams and to be aware that the two probabilities at each stage add to 1.

In a number of cases the situation described in the question seemed to have been overlooked or misunderstood as a denominator of 6 was often seen in the lower pair of branches in the completed tree diagram.

(a) Whilst many were able to complete the missing probability on the given branch there were many incorrect answers for the probabilities, and occasionally with the 'black' and 'white' labels, on the missing branches. Some even extended the tree further to the right.

(b) Solutions to this part showed that many candidates need to improve their knowledge of how the probabilities of combined events are calculated. It was not uncommon to see the addition (or subtraction) of two uncombined probabilities.

Those who did multiply two probabilities together often chose the two black probabilities from Bag B. Some calculated $\frac{3}{5} \times \frac{3}{7}$ only. Very few calculated $\frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times$ (probability of a black in the lower branch).

Answers: **(a)** $\frac{4}{7}$; $\frac{2}{7}$ and $\frac{5}{7}$ **(b)** $\frac{13}{35}$

Question 18

Answers to this question showed that many candidates need to improve their knowledge of the properties of angles in a circle. Many candidates made incorrect assumptions about the lines given in the diagram and gave a wide variety of incorrect answers.

(a) Those who realised that angles *CBE* and *CDE* are opposite angles in quadrilateral *BCDE*, and therefore supplementary, were the most successful.

(b) Those who realised that angles *BAC* and *BEC* are angles in the same segment were the most successful.

(c) This part was answered better by candidates as it only relied upon knowledge of alternate angles of the parallel lines *AB* and *ED*.

Answers: **(a)** 53 **(b)** 40 **(c)** 22

Question 19

This question was sometimes omitted, perhaps because of a lack of geometrical instruments, or perhaps because some candidates need to get a better understanding of loci and constructions.

(a) Some realised that it was necessary to draw the bisectors of the angles at *O*. Most of these, however, did not draw more than one or two angle bisectors, usually of the acute angles.

A common error was to draw the perpendicular bisector of either, or both, of the given lines *PQ* and *LM*.

Others constructed correct intersecting arcs but did not join the points of intersection. Some drew many arcs but chose to join the wrong points of intersection.

(b) This part was usually answered correctly.

Question 20

(a) Many candidates were able to find the midpoint of the two given points successfully. Common errors included the use of the gradient, or of the distance formula, or to subtract the coordinates rather than to add them. Some carelessness with signs, such as $-3 + 5 = -2$ or $-3 + 5 = -8$ was sometimes seen.

(b) Most candidates were able to find the gradient of the line joining the two points successfully. The common wrong answers were $\frac{3}{8}$, $-\frac{8}{3}$ or $\frac{5+(-3)}{1+4}$.

(c) This was a challenging question for many candidates, who did not seem to know where to start. Some tried to use equations, or gradients, of lines. Others tried to use the sum of vector components. The best attempts used the (Pythagorean) formula for the distance between two points and, provided the negative coordinates were manipulated correctly, usually obtained $OP = 5$ and $OR = 6$ and made the correct deduction.

Answers: **(a)** $(1,\ 2\frac{1}{2})$ **(b)** $-\frac{3}{8}$

Question 21

(a) This part was usually answered correctly, most candidates knowing that $9^1 = 9$ and $9^0 = 1$. The common wrong answers were 9 or 18.

(b) This part depended upon a candidate realising that $4^n = 2^{2n}$ and writing the equation $2n = n - 1$. Successful solutions were rather rare.

Answers: **(a)** 10 **(b)** –1

Question 22

(a) This part was answered well by most candidates. The usual mistake was to include the element 1.

(b) This was answered quite well, though some candidates need to be aware that n(*S*) means the number of elements in set *S*. Common wrong answers were {0, 2, 3, 4, 5, 6} or 4 or 1.

(c) Candidates who substituted the values 0, 1 and 2 for *x*, and 0 and 2 for *y*, into $2x + y$ dealt with this question competently, though careless errors in simple arithmetic occurred quite regularly.

Answers: **(a)** (0, 2) **(b)** 6 **(c)** 0, 2, 4, 6

Question 23

(a) This part was generally attempted successfully. Some wrong answers were caused by sign errors with the second vector; arithmetic errors with the negative numbers; multiplying the columns; using $2 \times 0 = 2$.

(b) Though many candidates obtained an answer with the correct order of the matrix, this question was found to be a challenge for some candidates. Arithmetic errors were quite common, particularly relating to the negative values. It was quite common to see a 2 by 3 matrix as the answer, and occasionally the matrix $\begin{pmatrix} -3 \\ -3 \\ 7 \end{pmatrix}$.

Answers: **(a)** $\begin{pmatrix} 5 \\ -9 \end{pmatrix}$ **(b)** $\begin{pmatrix} -3 & -3 & 7 \end{pmatrix}$

Question 24

Attempts at this question showed that some candidates need to get a better understanding of the properties of a speed-time graph, and to appreciate that the distance travelled during a given interval is obtained by finding the area under the graph for this interval.

(a) Most candidates made a reasonable attempt at this part and the correct graph was often drawn. The most common error was to draw the horizontal line from (30, 20) to (60, 20). Other errors were to draw a line from (0, 0) to (40, 20) or to draw a line from the right-hand end of the horizontal line to (120, 0).

(b) Candidates who knew that the distance travelled during a given interval is obtained by finding the area under the graph for this interval answered this part well. Some tried to find the distance by multiplying the total time by the constant speed. The error $20 \times 60 = 120$, or similar with 20×30 and 20×20, was sometimes seen.

Answers: **(a)** ruled line from (0, 0) to (30, 20) **and** ruled line from (30, 20) to (90, 20) **and** ruled line from (90, 20) to (110, 0) **(b)** 1700

Question 25

Some responses showed that some candidates need to gain a better understanding of vectors.

(a) This part was generally answered correctly. Not all answers were expressed in terms of **p**.

(b) Some candidates answered this part correctly. The common wrong answer was $\frac{1}{3}(3\mathbf{p} + 5\mathbf{q})$.

(c) Many candidates realised that it was necessary to add $2\mathbf{q}$ to the answer to part **(b)**.

(d) A minority of candidates equated their answer to part **(c)** to $k(\mathbf{p} + 3\mathbf{q})$ and hence found the value of k correctly.

Answers: **(a)** $3\mathbf{p}$ **(b)** $\frac{1}{2}(3\mathbf{p} + 5\mathbf{q})$ **(c)** $\frac{1}{2}(3\mathbf{p} + 9\mathbf{q})$ **(d)** 1.5

MATHEMATICS D

Paper 4024/21
Paper 2

Key messages

To succeed in this paper, candidates need to have studied the full content of the syllabus, including the new content, remember the necessary formulas and apply them appropriately. Candidates should use a suitable degree of accuracy in their working. Final answers should be rounded correct to three significant figures where appropriate or to the degree of accuracy specified in the question. Where candidates are required to show a result, they must set their work out logically and clearly, showing all stages of working leading to the required result.

General comments

The question paper consisted of a range of question types ranging from routine tasks to more complex problem-solving questions. Candidates of all abilities were able to demonstrate their understanding of a range of mathematical skills. Scripts covering the whole mark range were seen and candidates had enough time to complete the paper.

In general, candidates were able to access standard questions involving arithmetic, algebraic manipulation and graphs. Candidates found it harder to access questions where the process required needed to be identified, for example the use of basic trigonometry in **Question 6(c)** or Pythagoras' theorem in **Question 10(d)**. Some candidates lacked confidence in setting up algebraic expressions and manipulating them to form equations in **Questions 8** and **10(a)**.

In questions requiring candidates to show a result, key steps were sometimes omitted. Values should be shown substituted into formulae, rather than leaving this substitution to be implied, and the result should always be shown to at least one more significant figure than given in the question.

Candidates need to take care when using mathematical formulas. They were usually able to quote the cosine rule correctly, but errors were sometimes seen in the quadratic formula. The formulas for surface area and volume of a sphere were provided in **Question 9**, but many candidates did not realise that the question involved a hemisphere rather than a sphere and some candidates were unable to recall the relevant formulas for a cylinder.

In some places, candidates gave an inaccurate final answer due to inappropriate rounding of intermediate results. It is important that candidates retain sufficient figures in their working and only round to three significant figures for their final answer. Final answers rounded to two significant figures are not acceptable. Values of $\frac{22}{7}$ or 3.14 were sometimes used for π which also led to inaccurate final answers.

Comments on specific questions

Question 1

(a) (i) In general, candidates plotted the points correctly. In some cases, one or two points were slightly out of position due to lack of accuracy in plotting rather than misinterpretation of the scale.

(ii) Most candidates who understood the term correlation identified that it was positive correlation. In some cases, further information such as strong or weak was stated which was accepted. It was clear that some candidates had not met the term correlation as a range of irrelevant terms were also seen.

(iii) Many candidates identified the correct fraction, although in some cases it was not simplified as required by the question. Some candidates attempted to find a product of two fractions.

(iv) When a line of best fit is required, it should be ruled, have approximately the same number of points on either side of the line and cover the full range of data. Some candidates drew a good line of best fit and took an accurate reading from it. It was common to see lines drawn from corner to corner of the grid which was not acceptable. Some candidates did not understand the term 'line of best fit' and drew lines from point to point on the grid or omitted a line completely and made an estimate of the time.

(b) (i) Most candidates were able to identify the modal class, although some wrote down the frequency rather than the group interval.

(ii) Many candidates used the correct method to calculate the estimate of the mean. A common error was to use incorrect values for the midpoints, usually either the upper values from the ranges or 10.5, 30.5 and so on. Some candidates multiplied the frequencies by the group width rather than by the midpoint. Other errors were to add the midpoints and divide by 5 or add the frequencies and divide by 5.

(iii) Many correct answers were seen in this part. Some candidates divided the number of employees by 60 rather than 120 and others included the frequency for the $40 < t \leqslant 60$ group in their calculation.

Answers: **(a)(ii)** Positive **(iii)** $\frac{3}{5}$ **(b)(i)** $20 < t \leqslant 40$ **(ii)** 39.5 **(iii)** 22.5

Question 2

(a) Candidates who were able to work out that the length of time worked each day was 7 hours 45 minutes and could convert this correctly to 7.75 hours usually reached the correct answer. Many candidates had difficulty with the initial time calculation and, even if they found the time as 7 hours 45 minutes, they were unable to convert this to a decimal of an hour and used 7.45 in their calculation. Some candidates found the pay for one day or seven days rather than for the five days worked and others assumed that she was only paid for full hours worked.

(b) Many candidates were able to work out the percentage profit correctly; as the answer is exactly 23.75, this is the expected answer rather than a rounded value of 23.8 or 24. Some candidates divided by 495 rather than by 400 and others subtracted 19.80 from 400 instead of calculating the amount of money made by selling the 25 dresses.

(c) Candidates who understood that \$15.66 was 108% of the price excluding tax usually reached the correct answer. However, it was more common to see candidates reducing \$15.66 by 8% rather than using the correct reverse percentage method.

(d) Candidates who were familiar with the compound interest formula often applied it correctly and reached the correct answer. The question asked for the answer correct to the nearest cent, but many candidates rounded their answer to the nearest 10 cents or nearest dollar and so were not given full credit, even though the calculations had been carried out correctly. Some candidates started with the correct formula but evaluated it in stages and rounded prematurely thus reaching an inaccurate final answer. It was common to see calculations such as $3500 \times 1.017^4 = 3500 \times 1.07$ which would have given the correct answer if the initial expression had been evaluated on the calculator, rather than using 1.017^4 rounded to three significant figures in the calculation. Some candidates knew the compound interest formula and substituted values in the wrong positions and many worked with simple interest rather than compound interest.

Answers: **(a)** 395.25 **(b)** 23.75 **(c)** 14.50 **(d)** 3744.14

Question 3

(a) Candidates who identified that they needed to use the cosine rule to find the angle often showed correct working leading to the answer. In order to gain full credit, they needed to show their answer to at least one more significant figure than given in the question to demonstrate that they had evaluated it correctly. Some candidates attempted to use the sine rule or to use right angled trigonometry which were not suitable approaches.

(b) Some candidates showed clear and correct working in this part leading to a correct answer, however there were many errors seen in all stages of the calculation. Some were not able to calculate the area of the triangle correctly and often used 132 in place of 174 in the area formula. There was confusion about whether to multiply or divide by 3 to find the number of grams of seed needed, but most candidates were able to deal with $8.50 for 100 grams correctly. Some candidates did not start by calculating an area but tried to manipulate only the numbers given in this part of the question to get their result.

Answers: **(b)** 1580

Question 4

(a) (i) Candidates were often able to enter some of the numbers into the Venn diagram correctly, but common errors were to insert 1 outside of set *A*, to only complete sets *A* and *B*, to repeat numbers in more than one subset or to include values greater than 10.

(ii) Many candidates interpreted the set notation correctly here and gave the correct answer following through from their Venn diagram.

(iii) It was more common to see candidates describing the set notation in words rather than describing the elements in $A \cap B'$. A description of the notation using correct mathematical language was given credit, but it should be noted that the term 'prime' is not equivalent to the term 'complement'. Some candidates were clearly confused between the symbols for intersection and for union.

(b) (i) Many candidates found it difficult to interpret the information and draw a correct Venn diagram. It was common to see 12 taken as the number of people who liked oranges only rather than including those who liked bananas as well; this error led to the common incorrect answer of 5 people. Candidates using an algebraic approach often made the same error and set up the equation $5 + 12 + 8 + x = 30$, again leading to the answer 5.

(ii) Candidates found this part of the question challenging. Some were able to identify the correct probabilities to multiply and gave an answer that followed through correctly from their Venn diagram. Others did not appreciate that the probability was without replacement so calculated $\frac{7}{30} \times \frac{7}{30}$. It was common to see attempts to add probabilities or the probability of a single event rather than a combined event found.

Answers: **(a)(ii)** 6 **(iii)** Factors of 10 **(b)(i)** 10 **(ii)** $\frac{42}{870}$

Question 5

(a) Most candidates were able to calculate the value at $x = 4$ correctly.

(b) Most candidates plotted the points correctly and joined them with an acceptable smooth curve. When candidates made errors in plotting, the point (–1, – 0.6) was often plotted at (–1, 0.6).

(c) Candidates usually attempted to draw a tangent at the correct position on the curve. Some good tangents were seen that touched the curve at $x = -2$ but some lines remained in contact with a long section of the curve and were not centred on $x = -2$. When calculating the gradient, some candidates omitted the negative sign and others misread the scale on the vertical axis leading to a gradient outside of the acceptable range.

(d) This part was challenging to many candidates. The first step was to rearrange the given equation to identify that the solution would be found from reading the graph at $y = 2$. Only a minority of candidates were able to do this. Those candidates who read the graph at the correct point often only gave one or two of the required three solutions.

Answers: **(a)** –1.6 **(b)** Correct curve **(c)** –3.1 to –2.2 **(d)** –2.4 and 1.5 and 2.8

Question 6

(a) In a question asking to show that triangles are congruent, candidates are expected to state three appropriate pairs of sides or angles with geometrical reasons and state the condition for congruence they have used. Very few candidates were able to complete this successfully. Many were able to identify two or three correct pairings, but they often omitted reasons or did not give sufficient detail. Reference to angles *TAO* and *TCO* being equal needed to include the fact that the tangent is perpendicular to the radius. Reference to *AT* being equal to *CT* needed to include the fact that tangents from an external point are equal. Many candidates referred to the symmetry of the shape, which can only be established by first demonstrating congruence. It was common to see confusion between congruent triangles and similar triangles; it is not possible to establish congruence by referring to equal angles only.

(b) Most candidates had difficulty in finding expressions for the required angles in this part. It was rare to see a correct expression for angle *AOT*, and if that had not been found correctly, few candidates were able to go on and find either of the other two angles. The question asked for angles in terms of x, but many answers were purely numerical or given in terms of the vertices of the shape such as angle *OAT* = angle *OCT*.

(c) Candidates who identified that they needed to use right-angled trigonometry in triangle *OAT* to find *OT* first usually went on to reach the correct answer for *BT*. Some answers were inaccurate due to candidates evaluating sin 35 and using this value rounded to three significant figures in their calculation rather than directly calculating 6 ÷ sin 35. Many candidates did not identify the need to use trigonometry and attempted to use calculations involving arc length or sector area.

Answers: **(a)** Correct congruence proof **(b)(i)** $90 - x$ **(ii)** $\frac{90 - x}{2}$ **(iii)** $270 - x$ **(c)** 16.5

Question 7

(a) Many candidates were able to find the correct position vector of point *B*. The most common error was to calculate $\pm\left(\overrightarrow{OA} - \overrightarrow{AB}\right)$ rather than $\overrightarrow{OA} + \overrightarrow{AB}$.

(b) Many candidates were able to find the magnitude of *AB* correctly. A common error was to make an error with the negative sign and calculate $6^2 - 3^2$ rather than $6^2 + (-3)^2$. Some candidates did not understand the notation used for magnitude and gave a vector as an answer.

(c) Some candidates were able to start by finding the correct vector $\overrightarrow{CB}$, but they then often were unable to use this with their position vector of *B* to find the coordinates of point *C*. A common error was to multiply, rather than divide, $\overrightarrow{AB}$ by 3 to find $\overrightarrow{CB}$.

(d) (i) The first stage required in the solution was to use vector *AB* to find the gradient of the line *L* as $-\frac{1}{2}$. Few candidates were able to do this successfully, and they often attempted a calculation combining this vector with the coordinates of the given point leading to a gradient of 1 or –1. Many of them, however, were able to use the coordinates (–2, 5) in an equation $y = mx + c$ with their gradient to find the value of *c* and gained partial credit.

(ii) In this part, candidates were required to know that the product of the gradients of equations of perpendicular lines is –1. Some were able to apply this to the gradient of their answer to part **(i)** and give a correct equation. Many candidates did not appreciate that as line *M* passed through the origin, the equation would be of the form $y = mx$ and the addition of a constant was often seen.

Answers: **(a)** $\begin{pmatrix}2\\4\end{pmatrix}$ **(b)** 6.71 **(c)** $(0, 5)$ **(d)(i)** $y = -\frac{1}{2}x + 4$ **(ii)** $y = 2x$

Question 8

(a) Many candidates were able to complete the rectangle with the correct expressions. A small proportion of candidates entered numbers rather than expressions.

(b) Candidates who had the correct expressions in part **(a)** were often able to use these to write correct expressions for the products. Some errors were seen in the expansion of $(n + 5)^2$ and errors in signs were common when subtracting the two expressions so the difference of 25 was rarely reached with full correct working. Some candidates attempted to demonstrate the result by selecting values from the rectangle, but this was not given credit as it did not demonstrate that the difference was always 25.

(c) Candidates with the correct algebraic expressions were often able to set up an equation and solve it to find the required value. Some did not read the question carefully and gave the smallest number in the rectangle rather than the largest number. Some candidates did not use an algebraic approach and successfully solved the problem using a trial and improvement approach. It was common to see the incorrect answer 58 resulting from 174 ÷ 3.

Answers: **(a)** $n + 5$, $n + 10$ **(b)** Result correctly derived **(c)** 63

Question 9

(a) (i) This question asked for the result to be shown, so clear working was expected with substitutions into the correct volume formulae shown. The best approach was to show a calculation for the volume of the cylinder with the correct radius and height substituted and a calculation for the volume of the hemisphere with the correct radius substituted and then add these two results, giving the answer correct to four significant figures. In many cases the working was unclear, and the correct formulae were not used. It was common to see the volume formula for a sphere rather than a hemisphere used, for 24 to be used as the height of the cylinder rather than 21 or for the volume formula for the cylinder to be quoted incorrectly. Some candidates worked back from the given result to attempt to find the height of the cylinder rather than identifying that it was 3 less than the total height of the solid.

(ii) Candidates found this part of the question challenging and the correct result was rarely seen. Many were unable to recall the formula for the surface area of a cylinder and some calculated the surface area of a sphere rather than a hemisphere. It was common to see the surface area of the cylinder omitted, perhaps because it was not considered to be part of the curved surface. Some candidates added the area of the base of the solid, even though the question asked for just the curved surface area.

(iii) This question required candidates to use the volumes given in the question to find the linear scale factor as $\sqrt[3]{\frac{450}{650}}$ and use this to find the height of lamp *B*. The most common error was to use $\frac{450}{650}$ as the linear scale factor. Some candidates identified the correct relationship between the scale factors but were unable to calculate the height correctly. Some attempted to use the given values in a volume formula and rearrange this to find the height.

(b) Candidates found the use of bounds in a calculation very challenging. Many candidates converted both masses to a common unit and subtracted the mass of eight lamps from the mass of the case, but they either ignored the bounds completely or attempted to deal with them at the end of their calculation. Those who did start by using the bounds to find the values to use in their subtraction usually attempted to use the upper bound of both values. The correct method to find the upper bound of the mass is to subtract of the lower bound of the mass of the eight lamps from the upper bound of the mass of the case.

Answers: **(a)(i)** Result correctly derived **(ii)** 452 **(iii)** 21.2 **(b)** 1.57

Question 10

(a) Many candidates identified that the height of the prism was $x - 4$, but they were not always able to show the correct equation for the volume of the prism. Those who did attempt to set up the volume equation often used the volume of a cuboid rather than a triangular prism. Candidates who set up the correct equation were usually able to rearrange it correctly to the required form. Some candidates attempted to work back from the given equation or to solve the equation in this part of the question.

(b) Candidates who were able to quote the quadratic formula correctly were often able to substitute correctly into it and reach the correct solutions. Common errors were to omit negative signs, to calculate -12^2 instead of $(-12)^2$, or to use a short division line in the formula. Most candidates gave their solutions to the required accuracy.

(c) Candidates were often able to follow through from their solutions in part **(b)** to find the height of the prism. Some attempted to find the height by substituting into a volume formula rather than identifying that the height would be 4 less than their positive solution in part **(b)**.

(d) The use of trigonometry in three dimensions was challenging for many candidates. Some were able to use Pythagoras' theorem correctly to find either *AF* or *BF*, but then were unable to identify the correct triangle to use to find angle *AFB*.

Answers: **(a)** $3x^2 - 12x - 176 = 0$ correctly derived **(b)** 9.92 and –5.92 **(c)** 5.92 **(d)** 18.2

Question 11

(a) Many candidates answered this part correctly. The most common error was to start by writing the fractions correctly with a common denominator but make a sign error when expanding $4(x - 2) - 3(2x - 3)$. Candidates who expanded the denominator usually did this correctly.

(b) Many candidates were able to factorise the expressions in the numerator and denominator correctly and simplify to give the correct answer. A small number attempted to further simplify once the correct answer had been reached. Where errors in factorisation were seen, it was usually with the denominator rather than the difference of two squares in the numerator. Some candidates attempted to cancel terms in the numerator and denominator instead of starting by factorising the expressions.

Answers: **(a)** $\dfrac{1-2x}{(2x-3)(x-2)}$ **(b)** $\dfrac{2x-3}{x-5}$

MATHEMATICS D

Paper 4024/22
Paper 2

Key messages

To succeed in this paper, candidates need to have studied the full content of the syllabus, including the new content, remember the necessary formulas and apply them appropriately. Candidates should use a suitable degree of accuracy in their working. Final answers should be rounded correct to three significant figures where appropriate or to the degree of accuracy specified in the question. Where candidates are required to show a result, they must set their work out logically and clearly, showing all stages of working leading to the required result.

General comments

Most candidates demonstrated a good grasp of the topics, including those topics which are new to this syllabus. The most common way candidates, who understood the methods for certain questions, did not score full marks was by premature approximation resulting in final answers outside the acceptable range. Candidates need to ensure either accurate answers are kept and used in their calculator or intermediate working has values to a minimum of four significant figures. The candidates need to ensure they follow the instructions on the front of the exam paper which states that answers should only be rounded to three significant figures when the answer is not exact. For example, non-exact currency answers, in dollars, should be given to the nearest cent.

Comments on specific questions

Question 1

(a) Some of the candidates understood the requirements for this question by calculating 91% of 32500 followed by 78% of 29575. Occasionally candidates decided to round the exact money answer to 3 significant figures or to the nearest dollar, not following the instructions on the front of the paper. The most common mistake that candidates made was to calculate 31% of 32500, either in stages or as one calculation. Some candidates stated the total of his pension and his tax as their answer rather than the amount left after paying his pension and his tax.

(b) The formula for compound interest was seen correctly used on many scripts, however not all these completed the question correctly. Some candidates did not give the amount to the nearest cent as asked, while others did not appreciate that the formula gave the amount of money in the account at the end of 5 years and either added or subtracted 1200 to this amount. The mistakes seen in the compound interest formula included $1200\left(1-\frac{1.8}{100}\right)^5$ and $1200\left(1\times\frac{1.8}{100}\right)^5$. Many candidates used simple interest to calculate their answer.

(c) A minority of candidates scored full marks for this question. Very few candidates appreciated the meaning of the line ‘At the end of 5 years there was $828.75 in this account’. Many of the candidates based their calculation on $828.75 being the interest at the end of the 5 years.

(d) Candidates had most success with this part of the first question. A common mistake was to change £79.20 into dollars and then change that value into euros, as opposed to the remaining amount into euros. Other mistakes included converting $275 into pounds, subtracting £79.20, and then not knowing how to correctly convert from pounds to euros.

Answers: **(a)** 23068.50 **(b)** 1311.96 **(c)** 750 **(d)** 181.50

Question 2

(a) Many correct cumulative frequency graphs were seen, however there was some incorrect plotting of the point (120, 112). Some candidates plotted at the midpoint of the intervals. Other errors included using the information to plot a bar graph.

(b)(i) The majority of candidates correctly read the mass when the cumulative frequency was 100. Common wrong answers were 112 and $m \leqslant 120$, where candidates used the information from the middle column of the cumulative frequency table.

(ii) Correct methods were seen on many scripts with candidates accurately reading the upper and lower quartiles and subtracting one from the other. Mistakes included subtracting 50 from 150 and then reading the mass when the cumulative frequency was 100, with many candidates then giving the same answer as the median. Other mistakes were to give the answer as 100, to have errors due to inaccurate reading of the mass for either the upper or lower quartile, or both, or to read the upper or lower quartile at the wrong cumulative frequency, for example at 160.

(c) Many candidates realised that a comment was required in relation to the median and a comment relating to the interquartile range. The better responses made a clear comparison, appreciating the need to interpret what the interquartile range measures.

(d)(i) Most candidates were able to correctly write down the missing two values of the table. Some candidates left the rows blank, others thought that the other 72 should be divided equally between the two rows.

(ii) About a half of the candidates knew how to state the modal class from a grouped frequency table. The most common mistake was to write down the frequency of 64. Those who left the rows of the table blank usually did not attempt to state the modal class.

(iii) Candidates usually showed the method they had used and some fully correct answers were seen. Some candidates made a slight error with one of the values, or made errors by not using the midpoints, or did not divide by 200. Another common error was to multiply the frequencies by the class width and not by the midpoint.

Answers: **(b)(i)** 119 **(ii)** 16 **(d)(i)** 46, 26 **(ii)** $110 < m \leqslant 120$ **(iii)** 118.8

Question 3

(a) Many candidates demonstrated a correct method to simplify the algebraic fractions. The most common mistake was made in the numerator when expanding $-5(y-1)$. Some mistakes were seen in the denominator, usually by either incorrectly expanding $(y-1)(y+6)$ or incorrectly writing the denominator as $y - 1 + y + 6$.

(b) Several candidates were able to correctly factorise both the numerator and the denominator and then correctly simplify the fraction. Occasionally mistakes were seen in factorising the numerator, usually to $(2v-3)(v+3)$, or incorrectly factorising the denominator, usually to $(v-4)^2$. Some candidates did not attempt any factorisation and assumed the fraction could be simplified by cancelling individual terms in the numerator and the denominator.

(c) Most candidates correctly expanded the equation to $3x^2 + 9 = 11x$. However, some did not know how to proceed to solve an equation of this form. Of those who proceeded to have a quadratic equation set equal to 0, many left it as $3x^2 + 9 - 11x = 0$ before attempting to use the quadratic formula. It was not uncommon to find that b was substituted as 9 and c as –11 in this formula. Those candidates who correctly substituted normally went on to find both solutions correct to 2 decimal places. Occasionally '$-b$' was seen as –11 or b^2 was seen as –121. Working was usually seen to support the answers but, in a few cases, wrong working was seen leading to the correct answers, presumably from use of a calculator to solve the quadratic. On some occasions a wrong quadratic formula was quoted and used.

Answers: **(a)** $\dfrac{23-2y}{(y-1)(y+6)}$ **(b)** $\dfrac{2v+3}{v+4}$ **(c)** 2.43, 1.23

Question 4

(a) Some candidates knew how to calculate the average speed for the whole journey. The most common error was to calculate the speed for the remaining 20 miles and either give the answer as the total of the two speeds or the average of the two speeds.

(b) Some understanding of bounds was demonstrated by the majority of candidates with many correctly giving the upper bound for the distance. Not many appreciated that the upper bound for the speed is found using the upper bound for the distance and the lower bound for the time, although they usually knew how to calculate the average speed. Occasionally candidates attempted to adjust the upper bound after calculating the speed from the given values.

(c) Few fully correct answers were seen in this part. Many who appreciated the need to multiply two probabilities treated the two events as dependent, resulting in $\frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$. Other common mistakes seen were to add the two correct probabilities rather than multiply them or to give the answer $\frac{3}{8}$, the probability she is late on one of the days and not on both.

Answers: **(a)** 49.7 **(b)** 45.6 **(c)** $\frac{9}{64}$

Question 5

(a) Many chose to use the formula $\frac{1}{2}ab\sin C$ to correctly calculate the area of the triangle. Occasionally those using this formula wrote *ab* as 12. Some candidates chose to work out a length and perpendicular height for the triangle, but some used premature approximation, resulting in an inaccurate area. Some candidates calculated the area of sector *AOB* rather than the triangle.

(b) About half of the candidates were able to correctly find the area of sector *AOD*. Mistakes included finding the area of sector *AOB* or triangle *AOD*. Occasionally candidates did not use the specified values of π given on the front of the paper.

(c) Candidates obtaining full marks in this part were in the minority. Some candidates who correctly calculated the shaded area then gave this as a percentage of the area of the rectangle. Other candidates made mistakes calculating the shaded area but knew how to give this as a percentage of the area of the circle.

Answers: **(a)** 13.8 **(b)** 15.7 **(c)** 27.8

Question 6

(a) Some candidates were able to show the required equation**.** Some candidates used *y* as the length of the vegetable garden instead of the total length of the fencing.

(b) (i) The table was completed correctly by most candidates.

(ii) Accurately drawn curves were seen on the majority of scripts. A minority of candidates either used straight lines to join the points rather than a curve or mis-plotted some points.

(c) At least one correct answer was seen on many scripts, but some chose to give the second value very near to the first, for example 1.6 and 1.8. Occasionally candidates read their graph at 16 m or 14.5 m.

(d) (i) Better responses recognised that the minimum cost will occur when the minimum number of metres of fencing is used. Some candidates found the cost when 14 m was used, linking their response to this part with part **(c)**.

(ii) Many candidates understood that the amount of fencing required in this part was 17.5 m, but not all of these candidates were able to use that information on the graph to work out the greatest possible width of the garden. A few candidates used the graph, however they found the least possible width.

Answers: **(b)(i)** 20, 13, 20 **(c)** 1.7, 5.3 **(d)(i)** 240 **(ii)** 7.5

Question 7

(a) (i) Some candidates were able to correctly draw the enlargement, using the matrix to calculate the new vertices, while others using their knowledge of the transformation which this matrix represents. The majority of candidates drew a triangle that was a translation, rotation or reflection of triangle *A*.

(ii) There were many correct answers to this part, some having drawn the correct triangle *B* and some not. Not all those who knew it was enlargement gave both the scale factor and the centre. Some candidates wrote two transformations.

(iii) Some correct answers were seen, but many gave the answer 1:2.

(b) Correct answers were seen of a reflection of triangle *B* in the *x*-axis. Some candidates incorrectly reflected in the *y*-axis or reflected the wrong triangle.

(c) A correct matrix was seen on only a minority of scripts. A few gave the answer as $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, having omitted the determinant.

Answers: **(a)(iii)** 1:4 **(c)** $\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

Question 8

(a) (i) Some candidates were able to correctly draw the given line. However, some did not draw this across the whole grid. It was common to see the lines $y = x + 2$ or $x + y = 2$ drawn.

(ii) The correct region indicated was seen on many of the scripts.

(b) More efficient responses recognised the need to find the gradient of the line $2y = x + 4$ and used this to find the gradient of the perpendicular line. Many used the point (1, 8) and another point on the line *L* to show the gradient is –2. Having obtained a gradient, many knew how to use this along with a given point to obtain the equation of the line.

(c) Many correct answers were seen here. Better responses used the substitution method; errors were more common from candidates attempting to use elimination. Occasionally candidates incorrectly attempted to use one equation to obtain a value for *x* and the other equation to find a value for *y*, each requiring an assumption for the value of the other variable.

Answer: **(c)** (8.4, 6.2)

Question 9

(a) Correct methods to show the height of the pyramid were seen on many scripts, normally using Pythagoras' Theorem to find *AC* or *AE* and then Pythagoras' Theorem again to find *EF*. Some candidates incorrectly used a circular argument, for example assuming *EF* is 8.72 to obtain *AE* and then using this value of *AE* to obtain *EF*.

(b) The majority of the candidates used the correct formula for the volume of a pyramid, however occasionally candidates used $\frac{1}{2}$ instead of $\frac{1}{3}$. A common misunderstanding was to use the slant height instead of the perpendicular height.

(c) Of those candidates who knew how to find the required angle, the cosine rule was the most popular method, most starting with the explicit form for finding an angle. Some who used the implicit form made mistakes when rearranging the formula.

(d) The majority of candidates did not visualise the required angle and consequently inappropriate working was seen, many using working involving 9.5. Those who used indirect methods did not always obtain accurate answers. Misunderstanding was sometimes seen with candidates obtaining the angle of depression rather than the angle of elevation.

Answers: **(b)** 77.5 **(c)** 38.1 **(d)** 76.1

Question 10

(a) Many candidates were able to state at least one pair of angles that were the same but often no reasons or incorrect reasons were given. The most common correct reason stated was for the vertically opposite angles. Many gave other reasons involving alternate angles, which needed an assumption about parallel lines to make that conclusion. Very few candidates were able to correctly prove, with reasons, that the triangles were similar.

(b) (i) Many candidates gave their answer as a number rather than an expression, in terms of x.

(ii) Candidates obtaining the correct expression here were in the minority.

(c) Candidates had most success with this part of the question, correctly demonstrating how to find a side using a length of a similar triangle. Common mistakes included assuming that CY was 5.6 or that the length AE was found by the sum of the three given lengths.

Answers: **(b)(i)** $90 - x$ **(ii)** $180 - 2x$ **(c)** 16.64